

XII STANDARD
MODEL EXAM – 2021
MATHS

TIME : 3.00 hours

MARKS : 90

PART – I

I ALL QUESTIONS ARE COMPULSORY

20 x 1 = 20

- 1 If $|\text{adj}(\text{adj } A)| = |A|^9$, then the order of the square matrix A is.

(1) 3 (2) 4 (3) 2 (4) 5

- 2 If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

1. $\begin{bmatrix} 2 & -5 \\ -3 & 8 \\ 8 & -5 \\ -3 & 2 \end{bmatrix}$ 2. $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ 3. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ 4.

- 3 If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then K = -

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

- 4 Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 \\ 1 & 3 \\ -1 & 1 \end{bmatrix} \begin{matrix} -1 \\ x \\ 3 \end{matrix}$

If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

- 5 If $A(\text{adj } A) = \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}$ then $|A| =$

(1) 0 (2) 15 (3) -15 (4) $\frac{-15}{2}$

- 6 The conjugate of a complex number is $\frac{1}{i-2}$ then, the complex number is

(1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

- 7 If $1 \leq |Z - 2 + i| \leq 2$, then the greatest value of $|Z|$ is

(1) $\sqrt{3} - 2$ (2) $\sqrt{3} + 2$ (3) $\sqrt{5} - 2$ (4) $\sqrt{5} + 2$

- 8 If $(1+i)(1+2i)(1+3i) \dots (1+ni) = x + iy$, then $2.5.10 \dots (1+n^2)$ is

(1) 1 (2) i (3) $x^2 + y^2$ (4) $1 + n^2$

- 9 If $x^2 + y^2 = 1$, then $\frac{1+x+iy}{1+x-iy} = \text{-----}$
 (1) $x - iy$ (2) $2x$ (3) $-2iy$ (4) $x + iy$
- 10 If $z_1 = 1 - 2i$ and $z_2 = -4 + 2i$ then $z_1 + z_2 =$
 (1) $5 + 4i$ (2) $-5 - 4i$ (3) -3 (4) -5
- 11 A zero of $x^3 + 64$ is
 (1) 0 (2) 4 (3) $4i$ (4) -4
- 12 If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is
 (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$
- 13 The number of positive zeros of the polynomial $\sum_{j=0}^n n C_r (-1)^r x^r$ is
 (1) 0 (2) n (3) $< n$ (4) r
- 14 If $7x^3 - 43x^2 = 43x - 7$ has one zero 7, then find other two zeros
 (1) $1, \frac{1}{7}$ (2) $-1, \frac{-1}{7}$ (3) $-1, \frac{1}{7}$ (4) $1, \frac{-1}{7}$
- 15 $\sin^{-1} \frac{13}{5} - \cos^{-1} \frac{12}{13} + \sec^{-1} \frac{5}{3} - \operatorname{cosec}^{-1} \frac{13}{12}$ is equal to
 (1) 2π (2) π (3) 0 (4) $\tan^{-1} \frac{12}{65}$
- 16 If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in \mathbb{R}$, the value of $\tan^{-1} x$ is
 (1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$
- 17 If $\cot^{-1}(\sqrt{\sin \alpha}) + \tan^{-1}(\sqrt{\sin \alpha}) = \mu$, then $\cos 2\mu$ is equal to
 (1) $\tan^2 \alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$
- 18 $\sin(\tan^{-1} x)$, $|x| < 1$ is equal to
 (1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{x}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$
- 19 $\tan(\tan^{-1}(\frac{5\pi}{4})) =$
 (1) $\frac{5\pi}{4}$ (2) $-\frac{5\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{4}$
- 20 The principal domain of $\sin^{-1} x$ is
 (1) $[0, \pi]$ (2) $[\frac{-\pi}{4}, \frac{\pi}{4}]$ (3) $\mathbb{R}$ (4) $[-1, 1]$

PART – II

ANSWER ANY SEVEN QUESTIONS

7 x 2 = 14

QUESTION NUMBER 30 IS COMPULSORY

- 21 Find the rank of $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1^1 & -1^{-2} & 1^3 \end{bmatrix}$
by row reduction method
- 22 Find the smallest positive integer 'n' for which $\left[\frac{1+i}{1-i}\right]^n = 1$
- 23 If $z_1 = 3-2i$ and $z_2 = 6 + 4i$, find $\underline{Z_1}$
 $\underline{Z_2}$
in the rectangular form.
- 24 Simplify $\sum_{n=1}^{102} i^n$
- 25 If n and q are the roots of the equation $lx^2 + nx + n = 0$ show that
 $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$
- 26 Solve the equation $x^4 - 14x^2 + 45 = 0$
- 27 Discuss the maximum possible number of positive and negative roots of the polynomial equation $x^{2021} + 2021x^{2020} + 1950x^{1947} - 1919 = 0$
- 28 Find the principal value of $\sin^{-1}(2)$, if it exists
- 29 Find all values of x such that $-6\pi \leq x \leq 6\pi$ and $\cos x = 0$
- 30 If A is symmetric, prove that adj A is also symmetric.

PART – III

ANSWER ANY SEVEN QUESTION

7 x 3 = 21

- 31 If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$
- 32 Find the rank of (i) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ (ii)
 $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$ by minor method
- 33 Show that $|3z - 5 + i| = 4$ represents a circle and find its centre and radius.
- 34 If $|z| = 3$, show that $7 \leq |Z + 6 - 8i| \leq 13$.
- 35 If P is real. Discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
- 36 Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has at least 6 imaginary solutions.

- 37 If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
- 38 Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.
- 39 Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$
- 40 Find real and imaginary part of $z = \frac{3i^{20} - i^{19}}{2i - 1}$

PART – IV

ANSWER ALL THE QUESTION

7 x 5 = 35

- 41(a) If $A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ prove that $A^{-1} = A^T$
- (or)
- 41(b) If $\text{adj}(A) = \frac{1}{a} \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ find A and A^{-1}
- 42(a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find the product AB and BA and hence solve the system of equation $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$
- (or)
- 42(b) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40 % acid solution (use Cramer's rule)
- 43 The upward speed v of rocket at time t is approximated by
- (a) $V(t) = at^2 + bt + c$, $0 \leq t \leq 100$ where a, b and c are constant. It has been found that the speed at time $t = 3$, $t = 6$, and $t = 9$ seconds are respectively 64, 133 and 208 miles per second respectively. Find the speed at time $t = 15$ seconds (use Gaussian elimination method)
- (or)
- 43(b) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions
- 44(a) If $z = x + iy$ is a complex number such that $\text{im}\left(\frac{2z+1}{1z+1}\right) = 0$ show that the locus of Z is $2x^2 + 2y^2 + x - 2y = 0$
- (or)
- 44(b) Let Z_1, Z_2 and Z_3 be complex numbers such that $|Z_1| = |Z_2| = |Z_3| = r > 0$
- And $Z_1 + Z_2 + Z_3 \neq 0$ Prove that $\left| \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} \right|$

45(a) If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation
 $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$
 (or)

45(b) Determine k and solve the equation
 $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of other two roots.

46 Solve the following equation

(a) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$
 (or)

46 Find the domain of the following

(b) $g(x) = 2\sin^{-1}\frac{x^2+1}{2x} - \pi/4$

47(a) Find the value of $\cos^{-1}$

(i) $\cos [\cos^{-1}(4/5) + \sin^{-1}(\frac{4}{3})]$

(ii) $\cos^{-1} [\cos(\frac{4\pi}{3}) + \cos^{-1}(\frac{5\pi}{3})]$

(or)

47(b) Find the value of

(1) $\tan(\cos^{-1}(\frac{1}{2})) - \sin^{-1}(\frac{-1}{2})$

(2) $\sin(\tan^{-1}(\frac{1}{2})) - \cos^{-1}(\frac{4}{5})$

(3) $\cos(\sin^{-1}(\frac{4}{5})) - \tan^{-1}(\frac{3}{4})$

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XII – MATHS QUESTION PAPER CORRECTIONS

QUESTION NO	UPDATED QUESTION
23	If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$ in the rectangular form
24	Simplify the following $\sum_{n=1}^{102} i^n$
34	If $ z = 3$, show that $7 \leq z + 6 - 8i \leq 13$.
41. (b)	If $\text{adj}(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A and A^{-1}
44. (b)	Let z_1, z_2 , and z_3 be complex numbers such that $ z_1 = z_2 = z_3 = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right = r$.
46. (b)	Find the domain of the following (i) $f(x) = \sin^{-1} \left(\frac{x^2 + 1}{2x} \right)$