XII STANDARD

MODEL EXAM – 2021

MATHS

TIME :3.00 hours **MARKS : 90**

PART – I

I **ALL QUESTIONS ARE COMPULSORY** **20 x 1 =20**

1 If $|\text{adj}(\text{adj } A)| = |A|^{9}$, then the order of the square matrix A is.

(1) 3 (2) 4 (3) 2 (4) 5

2 If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

1. $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ 2. $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ 3. $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$ 4. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

3 If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then K = –

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(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

4 Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$

If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

5 If $A(\text{adj } A) = \begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}$ then $|A|$ =

(1) 0 (2) 15 (3) –15 (4) $\frac{-15}{2}$

6 The conjugate of a complex number is $\frac{1}{i-2}$ then, the complex number is

(1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

7 If $|Z - 2 + i| \leq 2$, then the greatest value of $|Z|$ is

(1) $\sqrt{3} - 2$ (2) $\sqrt{3} + 2$ (3) $\sqrt{5} - 2$ (4) $\sqrt{5} + 2$

8 If $(1+i)(1+2i)(1+3i) \ldots\ldots (1+ni) = x + iy$, then $2.5.10 \ldots\ldots (1+n^2)$ is

(1) 1 (2) i (3) $x^2 + y^2$ (4) $1 + n^2$

9 If $x^2 + y^2 = 1$, then $\frac{1+x+iy}{1+x-iy}$ = ----------

(1) x − iy (2) 2x (3) −2iy (4) x + iy

10 If $z_1 = 1 - 2i$ and $z_2 = - - 4 + 2i$ then $z_1 + z_2$ =

(1) 5 + 4i (2) − 5 − 4i (3) −3 (4) − 5

11 A zero of $x^3 + 64$ is

(1) 0 (2) 4 (3) 4i (4) −4

12 If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

(1) $\frac{-q}{r}$ (2) $\frac{-p}{r}$ (3) $\frac{q}{r}$ (4) $\frac{-q}{p}$

13 The number of positive zeros of the polynomial $\sum_{j=0}^{n} nc_r (-1)^r x^r$ is

(1) 0 (2) n (3) < n (4) r

14 If $7x^3 - 43x^2 = 43x - 7$ has one zero 7, then find other two zeros

(1) 1, $\frac{1}{7}$ (2) −1, $\frac{-1}{7}$ (3) −1, $\frac{1}{7}$ (4) 1, $\frac{-1}{7}$

15 $\text{Sin}^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \text{cosec}^{-1}\frac{13}{12}$ is equal to

(1) 2π (2) π (3) 0 (4) $\tan^{-1}\frac{12}{65}$

16 If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

(1) $\frac{-\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $\frac{-\pi}{5}$

17 If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = \mu$, then $\cos 2\mu$, is equal to

(1) $\tan^2\alpha$ (2) 0 (3) −1 (4) $\tan 2\alpha$

18 Sin ($\tan^{-1} x$), $|x| < 1$ is equal to

(1) $\frac{x}{\sqrt{1-x2}}$ (2) $\frac{1}{\sqrt{1-x2}}$ (3) $\frac{x}{\sqrt{1+x2}}$ (4) $\frac{x}{\sqrt{1+x2}}$

19 $\tan(\tan^{-1}(\frac{5\pi}{4}))$ =

(1) $\frac{5\pi}{4}$ (2) $\frac{-5\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $\frac{-\pi}{4}$

20 The principal domain of $\sin^{-1} x$ is

(1) [0, π] (2) $[\frac{-\pi}{4}, \frac{\pi}{4}]$ (3) R (4) [−1, 1]

PART – II

ANSWER ANY SEVEN QUESTIONS 7 x2 = 14

QUESTION NUMBER 30 IS COMPULSORY

21 Find the rank of $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1^{1} & -1^{-2} & 1^{3} \end{bmatrix}$ by row reduction method

22 Find the smallest positive integer 'n' for which $\left[\frac{1+i}{1-i}\right]^{n} = 1$

23 If $z_1 = 3-2i$ and $z_2 = 6 + 4i$, find $\frac{Z_1}{Z_2}$ in the rectangular form.

24 Simplify $\sum_{n=1}^{102} i^{1}$

25 If n and q are the roots of the equation $lx^2 + nx + n = 0$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

26 Solve the equation $x^4 - 14x^2 + 45 = 0$

27 Discuss the maximum possible number of positive and negative roots of the polynomial equation $x^{2021} + 2021 x^{2020} + 1950 x^{1947} - 1919 = 0$

28 Find the principal value of $\sin^{-1}(2)$, if it exists

29 Find all values of x such that $-6\pi \leq x \leq 6\pi$ and $\cos x = 0$

30 If A is symmetric, prove that adj A is also symmetric.

PART – III

ANSWER ANY SEVEN QUESTION

7 x 3 = 21

31 If $F(\alpha) = \begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$

32 Find the rank of (i) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$ by minor method

33 Show that $|3z - 5 + i| = 4$ represents a circlur and find its centre and radius.

34 If $|z| = 3$, show that $7 \leq |Z + 6 - 8i| \leq 13$.

35 If P is real. Discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

36 Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

37 If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52cubic units. Find the volume of the cuboid.

38 Is cos −1 (−x) = π v− cos $^{-1}$ (x) true ? Justify your answer.

39 Find the value of $\sec^{-1}$ ($\frac{-2\sqrt{3}}{3}$)

40 Find real and imaginary part of z = $\frac{3i^{20}-i^{19}}{2i-1}$

PART – IV

ANSWER ALL THE QUESTION

7 x 5 = 35

41(a) If A = $\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ prove that $A^{-1} = A^{T}$

(or)

41(b) If adj (A) = $\frac{1}{a}\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ find A and A^{-1}

42(a) If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find the product AB and BA and hence solve the system of equation x − y + z =4 , x −2y−2z = 9, 2x + y + 3z = 1

(or)

42(b) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40 % acid solution (use Cramer's rule)

43 The upward speed vets of rocket at time t is approximated by

(a) V(t), = $at^2 + bt + c$, o ≤ t ≤100 where a,b and c are constant. It has been found that the speed at time t = 3, t=6, and t = 9 seconds are respectively 64,133 and 208 miles per second respectively. Find the speed at time t = 15 seconds (use Gaussion elimination method)

(or)

43(b) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions

44(a) If z = x + iy is a complex number such that im ($\frac{2z+1}{1z+1}$) = 0 show that the locus of Z is $2x^2 + 2y^2 + x - 2y = 0$

(or)

44(b) Zet Z1, Z2 and Z3 be complex numbers such that |Z1| = |Z2| = |Z3| = r> 0 And Z1 + Z2 + Z3 ≠ 0 Prove that $\left| \frac{z_1z_2+z_2z_3+z_3z_1}{z_1z_2+z_2z_3+z_3z_1} \right|$

45(a) If 2 + i and $3 - \sqrt{2}$ are roots of the equartion

$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$

(or)

45(b) Determine k and solve the equation

$2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of other two roots.

46 Solve the following equation

(a) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

(or)

46 Find the domain of the following

(b) $g(x) = 2\sin^{-1}\frac{x2+1}{2x} - \pi/4$

47(a) Find the value of $\cos^{-1}$

(i) $\cos[\cos^{-1}(4/5) + \sin^{-1}(\frac{4}{3})]$

(ii) $\cos^{-1}[\cos(\frac{4\pi}{3}) + \cos^{-1}(\frac{5\pi}{3})]$

(or)

47(b) Find the value of

(1) $\tan(\cos^{-1}(\frac{1}{2}) - \sin^{-1}(\frac{-1}{2}))$

(2) $\sin(\tan^{-1}(\frac{1}{2}) - \cos^{-1}(\frac{4}{5}))$

(3) $\cos(\sin^{-1}(\frac{4}{5}) - \tan^{-1}(\frac{3}{4}))$

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XII – MATHS QUESTION PAPER CORRECTIONS

QUESTION NO	UPDATED QUESTION
23	If $z_1 = 3-2i$ and $z_2 = 6+4i$, find $\frac{z_1}{z_2}$ in the rectangular form
24	Simplify the following $\sum_{n=1}^{102} i^n$
34	If $\lvert z \rvert = 3$, show that $7 \le \lvert z+6-8i \rvert \le 13$.
41. (b)	If $\text{adj}(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find $\lvert A \rvert$ and A^{-1}
44. (b)	Let z_1, z_2, and z_3 be complex numbers such that $\lvert z_1 \rvert = \lvert z_2 \rvert = \lvert z_3 \rvert = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left\lvert \frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3} \right\rvert = r$.
46. (b)	Find the domain of the following (i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$